

A generalization of Kolmogorov's theory of n-widths for infinite dimensional spaces: applications to Compressive sensing

Ognyan Kounchev

*Institute of Mathematics and Informatics
Bulgarian Academy of Sciences, Sofia, Bulgaria
kounchev@math.bas.bg*

Abstract

Recently, the theory of widths has got high interest due to its importance for the so-called Compressive sensing, see the works of D. Donoho, E. Candes, T. Tao, R. Baraniuk and others. On the other hand, the theory of n-widths of Kolmogorov (a kind of Optimal recovery theory) is not appropriate for the spaces of functions depending on several variables - this is seen from the simplest examples which one would like to have treated by a theory of Optimal recovery. We generalize the theory of n-widths of Kolmogorov by considering approximation by infinite-dimensional spaces of functions which have so far "harmonic dimension n" in some sense. A large class of such spaces having "harmonic dimension n" is provided by the solutions of elliptic differential operators of order $2n$. We indicate possible applications to Compressive sensing.