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HERMITIAN-LIKE NATURAL CONNECTION ON AN ALMOST CONTACT METRIC MANIFOLD

MILEN J. HRISTOV

ABSTRACT. On an almost contact metric manifold we introduce a natural connection which is an extension of the Hermitian connection on an almost Hermitian manifold. The objects, corresponding to such a connection, are described in terms of complex frame fields.

On an almost Hermitian manifold there exists a remarkable natural connection, i.e. a linear connection, preserving the almost Hermitian structure, the so called *Hermitian connection* [1,2]. In case of an almost contact metric manifold the class of the natural connections has been introduced in general by G.Ganchev, V.Alexiev [2]. In the present paper we deal with geometrically arising natural connection, whose restriction to the contact distribution coincides with the Hermitian connection. Such a connection we call a *Hermitian-like natural connection*. Analogously to [3,5], this connection and the objects related to it are described in terms of complex frame fields.

1. Preliminaries. Let M be a $2n+1$-dimensional differentiable manifold with an *almost contact metric structure* (φ,ξ,η,g), where φ is a tensor field of type (1,1), ξ is a vector field, η is a 1-form, g is a definite metric tensor field, such that

$$\varphi^2 = -\mathrm{id} + \eta\otimes\xi,\quad \varphi(\xi)=0,\quad \eta(\xi)=1,\quad \mathrm{g} = \mathrm{g}\circ\varphi + \eta\otimes\eta.$$

On such a manifold arise the operators $h = -\varphi^2, v = \eta\otimes\xi$, such that

$$h(T_pM) = \mathrm{Ker}\eta_p,\quad v(T_pM) = Im\eta_p \ \text{ and } \ T_pM = h(T_pM)\oplus v(T_pM) - \text{orthogonal}$$

and $U(n)\times 1$-invariant, $p\in M$. The complexification T_p^cM of the tangential space T_pM is decomposable as follows [5]:

$$T_p^cM = D_p^{10}\oplus D_p^{01}\oplus Im\eta_p,$$

where $D_p^{10}(D_p^{01})$ is $+i(-i)$-eigenspace of the operator φ_p and $D_p^c = D_p^{10} \oplus D_p^{01}$ is the complexification of the contact distribution $D_p = \mathrm{Ker}\eta_p, p \in$M. For any orhtonormal basis $\{e_\alpha, \varphi e_\alpha, \xi\}_{\alpha\in I}$, $I = \{1,2,\dots,n\}$ of T_pM, the vectors $Z_\alpha = e_\alpha - i\varphi e_\alpha$ (resp. $Z_{\bar\alpha} = \overline{Z_\alpha}$), $\alpha \in I$ form a basis for $D_p^{10}(D_p^{01})$, $Im\eta_p$ is spanned by $Z_o = \xi$ and so T_p^cM (resp. D_p^c) is spanned by the complex frame fields Z_A, $A \in I \cup \bar I \cup I_o$ (resp. Z_a, $a \in I \cup \bar I$), where $\bar I = \{\bar 1, \bar 2, \dots, \bar n\}$ and $I_o = \{0 = \bar 0\}$. Unless otherwise stated, Greek small letters will be run through the index-set I, Latin small- through $I \cup \bar I$ and Latin capital- through $I \cup \bar I \cup I_o$.

The structure (φ, ξ, η, g) is said to be *normal* if the tensor field $N = [\varphi,\varphi] + 2d\eta \otimes \xi$ of type (1,2) vanishes identically, where $[\varphi,\varphi]$ is the Nijenhuis tensor field, formed with

$$\varphi : [\varphi,\varphi](x,y) == [\varphi x, \varphi y] - \varphi[\varphi x, y] - \varphi[x, \varphi y] - \varphi^2[x,y], \quad x, y \in T_pM.$$

In that case $D_p^{10}(D_p^{01})$, is an involutive distribution [3]. The tensor field of type $(0,3)$ corresponding to N will be denoted by the same letter: $N(x,y,z) = g[N(x,y),z]$, $x,y,z \in T_pM$. When the structure (φ,ξ,η,g) is of *Hermitian type*, then $N = 0$ and $d\eta = 0$ are valid and for any point $p \in M$ there exists a local complex coordinate system $\{z^\alpha = x^\alpha + iy^\alpha, \overline{z^\alpha}, t\}_{\alpha\in I}$, such that $D_p^{10} = \mathrm{span}\{Z_\alpha = \partial/\partial z^\alpha = \partial_\alpha\}_{\alpha\in I}$, $D_p^{01} = \mathrm{span}\{Z_{\bar\alpha} = \partial/\partial\overline{z^\alpha} = \partial_{\bar\alpha}\}_{\alpha\in I}$ and $Z_o = \xi = \partial/\partial t$ [5].

Let $\Phi : \Phi(x,y) = g(x,\varphi y)$, $x,y \in T_pM$ be the fundamental 2-form on M, ∇ be the Levi-Civita connection of the metric g and $F = -\nabla\Phi$. The properties, formulated in the following Lemma are well known [2,5].

Lemma 1. *Let $M^{2n+1}(\varphi,\xi,\eta,g)$ be an almost contact metric manifold. Then for all $x,y,z \in T_pM$ the following equalities are true:*

$$(1) \qquad (\nabla\varphi)\circ\varphi = -\varphi\circ(\nabla\varphi) + (\nabla\eta)\otimes\xi + (\nabla_\xi)\otimes\eta;$$

$$(2) \quad F(x,y,z) = g[(\nabla_x\varphi)y,z] = -F(x,z,y) = -F(x,\varphi y,\varphi z) + [\eta\wedge(\nabla_x\eta)\circ\varphi](y,z);$$

$$(3) \qquad 2d\eta(x,y) = F(x,\varphi y,\xi) - F(y,\varphi x,\xi);$$

$$(4) \qquad 3d\Phi(x,y,z) = -\underset{(xyz)}{\mathcal{G}} F(x,y,z), \quad (\mathcal{G} - \textit{cyclic summation});$$

$$(5) \qquad N(x,y,z) = -3d\Phi(\varphi x,y,z) + 3d\Phi(\varphi y,x,z) + 2F(z,y,\varphi x) + \\ +2[\eta\otimes(\nabla\eta - 2d\eta)](x,y,z).$$

2. Hermitian-like natural connection.

Definition 1. [2,4] A linear connection D on an almost contact metric manifold $M(\varphi,\xi,\eta,g)$ is said to be *natural* if $Dg=0$, $D\varphi=0$.

If D is a natural connection, then $D\xi=0$ and $D\eta=0$ are also valid [2,4].

Using (1), it is easy to check the following

Lemma 2. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold. The linear connection $\overset{\circ}{D}$, defined by*

$$\overset{\circ}{D}_x y=\nabla_x y-\frac{1}{2}\varphi(\nabla_x\varphi)y+[(\nabla_x\eta)y]\xi-\frac{1}{2}\eta(y)\nabla_\xi x,$$

$x,y\in T_pM$ is natural.

We shall call $\overset{\circ}{D}$ a *basic natural connection.*

Theorem 1. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold. For any $\mathbb{R}$-valued differentiable functions λ,μ on M the linear connection $\overset{\lambda,\mu}{D}$, defined by*

$$g(\overset{\lambda,\mu}{D}_x y,z)=g(\overset{\circ}{D}_x y,z)+\frac{\lambda}{4}[3d\Phi(\varphi x,\varphi y,\varphi z)+3d\Phi(\varphi x,hy,hz)]+ \tag{6}$$

$$\frac{\mu}{4}\eta(x)[3d\Phi(\xi,\varphi y,hz)-3d\Phi(\xi,hy,\varphi z)],$$

$x,y,z\in T_pM$ is natural.

P r o o f. For any linear connection D on M there exists a uniquely determined tensor field Q of type (1,2) such that $D=\overset{\circ}{D}+Q$. Using Definition 1, it is clear that D is natural iff the tensor field of type $(0,3)$, corresponding to Q denoted by the same letter: $Q(x,y,z)=g[Q(x,y),z]$, has the symmetries

$$\left|\begin{array}{ll} Q(x,y,z)=-Q(x,z,y)=Q(x,\varphi y,\varphi z), & \\ Q(x,\xi,z)=Q(x,y,\xi)=0, & x,y,z\in T_pM. \end{array}\right. \tag{7}$$

It is well known that Q is decomposable, as follows

$$Q=hQ+vQ, \tag{8}$$

where hQ and vQ are mutually orthogonal with respect to the inner product in $\otimes^3T_p^*M$ induced from g and invariant under the action of the standard representation of the Lie-group $U(n)x1$ in $\otimes^3T_p^*M$. The defining conditions for the factors are:

$$(hQ)(x,y,z)=Q(hx,hy,hz),\quad (vQ)(x,y,z)=\eta(x)Q(\xi,hy,hz), \tag{9}$$

$x, y, z \in T_pM$.

Now (7) implies that the **C**-linear extension of $hQ(vQ)$ on T^c_pM is uniquely determined by the essential (i.e.nonvanishing in general) components $Q_{\alpha\beta\bar{\gamma}}(Q_{o\beta\bar{\gamma}})$ and also its conjugated ones.

The exterior differential $d\Phi$ of the fundamental 2-form is a geometric object on the manifold having analogous to (8) decomposition: $d\Phi = h(d\Phi) + v(d\Phi)$, where

$$h(d\Phi)(x, y, z) = d\Phi(hx, hy, hz),$$

$$v(d\Phi)(x, y, z) = \eta(x)d\Phi(\xi, y, z) - [\eta \wedge d\Phi(\xi, x, *)](y, z).$$

(here $d\Phi(\xi, x, *)y = d\Phi(\xi, x, y)$). The essential components of $h(d\Phi)$ are $\Phi_{\alpha\beta\bar{\gamma}}$, $\Phi_{\alpha\beta\gamma}$ and of $v(d\Phi)$ – $\Phi_{o\beta\bar{\gamma}}$, $\Phi_{o\beta\gamma}$. Therefore in the set of all tensor fields of type $(0,3)$, having the symmetries (7) there exists a 2-parameter family $\overset{\lambda,\mu}{Q}$ such that

$$\overset{\lambda,\mu}{Q}_{\alpha\beta\bar{\gamma}} = \frac{3}{2}i\lambda\Phi_{\alpha\beta\bar{\gamma}}, \quad \overset{\lambda,\mu}{Q}_{o\beta\bar{\gamma}} = \frac{3}{2}i\mu\Phi_{o\beta\bar{\gamma}}, \tag{10}$$

where λ and μ are $\mathbb{R}$-valued differentiable functions on M. Linearising the equalities (10), substituting α, β, γ with $x - i\varphi x, y - i\varphi y, z - i\varphi z$ respectively, using (7) and taking the real parts, we get

$$h\overset{\lambda,\mu}{Q}(x, y, z) = \frac{\lambda}{4}[3d\Phi(\varphi x, \varphi y, \varphi z) + 3d\Phi(\varphi x, hy, hz)],$$

$$\overset{\lambda,\mu}{Q}(\xi, y, z) = \frac{\mu}{4}[3d\Phi(\xi, \varphi y, hz) - 3d\Phi(\xi, hy, \varphi z)].$$

Substituting these expressions consequently in (9),(8), we get the 2-parameter family (6) of geometrical natural connections on M.

The next Lemma follows after direct computation.

Lemma 3 . *Let $M(\varphi, \xi, \eta, g)$ be an almost contact metric manifold. The torsion tensor field $\overset{\lambda,\mu}{T}$, corresponding to a geometrical natural connection $\overset{\lambda,\mu}{D}$, satisfies*

$$\overset{\lambda,\mu}{T}(x, y, z) = \frac{1}{2}F(x, y, \varphi z) - \frac{1}{2}F(y, x, \varphi z) + \frac{1}{2}[\eta \wedge (\nabla_* \eta)z](x, y) + 2d\eta(x, y)\eta(z) +$$

$$\frac{\lambda}{4}[6d\Phi(\varphi x, \varphi y, \varphi z) + 3d(\varphi x, hy, hz) + 3d\Phi(hx, \varphi y, hz)] +$$

$$\frac{\mu}{4}\{\eta \wedge [3d\Phi(\xi, \varphi *, hz) - 3d\Phi(\xi, h*, \varphi z)]\}(x, y),$$

$$x, y, z \in T_pM.$$

Remarks. 1) $\overset{\circ}{D} \equiv \overset{0,0}{D}$.

2) If $M(\varphi,\xi,\eta,g)$ is a Sasakian manifold, then $N = 0$, $\Phi = 2d\eta$ are valid and hence any geometrical natural connection $\overset{\lambda,\mu}{D}$ coincides with Tanaka's connection, whose torsion tensor field satisfies [2,4]

$$T(hx,hy,z) = d\eta(hx,hy)\eta(z),$$

$$T(\xi,hx,hy) + T(\xi,\varphi x,\varphi y) = 0.$$

3) The restriction of the connection $\overset{1,0}{D}$ to the contact distribution $D = \mathrm{Ker}\eta$, on which (φ, g) is an almost Hermitian structure, coincides with the Hermitian connection, whose torsion tensor field satisfies the so called "pure" property [1,3]:

$$T(hx,hy,hz) + T(\varphi x,\varphi y,hz) = 0.$$

Lemma 4 [2,4]. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold. For any natural connection D on M with torsion tensor field T, the next equalities are valid:*

a) $2d\eta(x,y) = \eta(T(x,y)) = T(x,y,\xi)$;

b) $3d\Phi(x,y,z) = T(x,y,\varphi z) + T(y,z,\varphi x) + T(z,x,\varphi y)$;

c) $N(x,y,z) = T(x,y,z) - T(\varphi x,\varphi y,z) - T(\varphi x,y,\varphi z) - T(x,\varphi y,\varphi z), x,y,z \in T_pM$.

Definition 2. A natural connection D on an almost contact metric manifold $M(\varphi,\xi,\eta,g)$ is said to be *Hermitian-like natural connection* if the torsion tensor field T, corresponding to D is "it pure", i.e. the next equalities are valid:

$$T(hx,hy,hz) + T(\varphi x,\varphi y,hz) = 0, \tag{11}$$

$$2[T(\xi,hy,hz) + T(\xi,\varphi y,\varphi z)] = T(hy,hz,\xi) + T(\varphi y,\varphi z,\xi), x,y,z \in T_pM. \tag{12}$$

Theorem 2. *On any almost contact metric manifold $M(\varphi,\xi,\eta,g)$ there exists uniquely determined Hermitian-like natural connection D. The defining conditions for D and for the torsion tensor field T corresponding to D are respectively*

$$g(D_xy,z) = g(\overset{\circ}{D}_xy,z) + \frac{1}{4}[3d\Phi(\varphi x,\varphi y,\varphi z) + 3d\Phi(\varphi x,hy,hz)] -$$

$$-\frac{1}{4}\eta(x)[3d\Phi(\xi,\varphi y,hz)-3d\Phi(\xi,hy,\varphi z)],$$

$$T(x,y,z)=\frac{1}{2}F(x,y,\varphi z)-\frac{1}{2}F(y,x,\varphi z)+\frac{1}{2}[\eta\wedge(\nabla_{*}\eta)z](x,y)+2d\eta(x,y)\eta(z)+$$
$$+\frac{1}{4}[6d\Phi(\varphi x,\varphi y,\varphi z)+3d\Phi(\varphi x,hy,hz)+3d\Phi(hx,\varphi y,hz)]-$$
$$-\frac{1}{4}\{\eta\wedge[3d\Phi(\xi,\varphi *,hz)-3d\Phi(\xi,h*,\varphi z)]\}(x,y),$$

$x,y,z\in T_pM$.

Proof. We shall show that $D\equiv\overset{1,-1}{D}$. Using Lemma 3 and (2), (4) we compute

$$\overset{\lambda,\mu}{T}(hx,hy,hz)+\overset{\lambda\mu}{T}(\varphi x,\varphi y,hz)=$$
$$=\frac{1}{2}F(hx,hy,\varphi z)-\frac{1}{2}F(hy,hx,\varphi z)+\frac{1}{2}F(\varphi x,\varphi y,\varphi z)-\frac{1}{2}F(\varphi y,\varphi x,\varphi z)+$$
$$+\frac{\lambda}{2}[3d\Phi(hx,hy,\varphi z)+3d\Phi(\varphi x,\varphi y,\varphi z)]=\frac{\lambda-1}{2}[3d\Phi(hx,hy,\varphi z)+3d\Phi(\varphi x,\varphi y,\varphi z)]$$

and so (11) is valid iff $\lambda=1$.

Further Lemmas 3,4 and (2),(3),(4) imply

$$2[\overset{\lambda,\mu}{T}(\xi,hy,hz)+\overset{\lambda,\mu}{T}(\xi,\varphi y,\varphi z)]=$$
$$(1+\mu)[(\nabla_{hy}\eta)hz+(\nabla_{hz}\eta)hy+(\nabla_{\varphi y}\eta)\varphi z+(\nabla_{\varphi z}\eta)\varphi y]+$$
$$\overset{\lambda,\mu}{T}(hy,hz,\xi)+\overset{\lambda,\mu}{T}(\varphi y,\varphi z,\xi),$$

and hence (12) is valid iff $\mu=-1$. From Lemma 3 we express the Hermitian-like torsion tensor field $\overset{1,-1}{T}$. □

3. Characterizations of the Hermitian-like natural connection in terms of local complex frame fields. We shall express the uniquely determined **C**-linear extension of the Hermitian-like natural connection and of the corresponding torsion tensor field, denoted by the same letters, in terms of the complex frame fields $\{Z_A\}_{A\in I\cup\bar{I}\cup I_0}$.

Lemma 5. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold. In terms of the complex frame fields $\{Z_A\}_{A\in I\cup\bar{I}\cup I_0}$ we have:*

i) the essential components of the Hermitian-like natural connection $D_{AB}^{C}=\overline{D_{\bar{A}\bar{B}}^{\bar{C}}}$ satisfy *a)* $D^{\gamma}_{\alpha\beta} = \Gamma^{\gamma}_{\alpha\beta} + \frac{3}{2}i\Phi^{\gamma}_{\alpha\beta}$,

b) $D^{\gamma}_{\bar{\alpha}\beta} = \Gamma^{\gamma}_{\bar{\alpha}\beta} - \frac{3}{2}i\Phi^{\gamma}_{\bar{\alpha}\beta}$,

c) $D^{\gamma}_{o\beta} = \Gamma^{\gamma}_{o\beta} - \frac{3}{2}i\Phi^{\gamma}_{o\beta}$, *where* Γ^{C}_{AB} *are components of the Levi-Civita connection;*

ii) the essential components of the Hermitian-like torsion tensor field $T_{ABC} = -T_{BAC} = \overline{T_{\bar{A}\bar{B}\bar{C}}}$ *satisfy*

a) $T_{\alpha\beta\gamma} = \frac{1}{4}N_{\alpha\beta\gamma}$,

b) $T_{\alpha\beta\bar{\gamma}} = 3i\Phi_{\alpha\beta\bar{\gamma}}$,

c) $T_{o\beta\gamma} = \frac{1}{2}N_{o\beta\gamma}$,

d) $T_{o\beta\bar{\gamma}} = \eta_{\beta\bar{\gamma}}$,

e) $T_{\alpha\beta o} = 2\eta_{\alpha\beta} = \frac{1}{2}N_{\alpha\beta o}$,

f) $T_{\alpha oo} = 2\eta_{\alpha o} = N_{\alpha oo}$.

The next Lemma follows immediately.

Lemma 6. *Let* $M(\varphi, \xi, \eta, g)$ *be an almost contact metric manifold of Hermitian type. Then:*

i) the essential components of the Hermitian-like natural connection are $D^{\gamma}_{\alpha\beta} = g^{\gamma\bar{\sigma}}\partial_{\alpha}g_{\beta\bar{\sigma}}$*;*

ii) the essential components of the Hermitian-like torsion tensor field are $T_{\alpha\beta\bar{\gamma}} = 3i\Phi_{\alpha\beta\bar{\gamma}}$.

Further, let $K = [D, D] - D_{[,]}$ be the curvature tensor field of type (1,3), corresponding to D and let the curvature tensor field of type (0,4) be denoted by the same letter:

$$K(x, y, z, u) = g[K(x, y)z, u], \quad x, y, z, u \in T_pM.$$

We shall call K a *Hermitian-like curvature tensor.*

The **C**-linear extension of K has the properties:

$$(13) \quad \left| \begin{array}{l} K(x, y, zu) = -K(y, x, z, u) = -K(x, y, u, z) = \overline{K(\bar{x}, \bar{y}, \bar{z}, \bar{u})}, \\ K(x, y, \varphi z, \varphi u) = K(x, y, z, u), \quad z, u \in D^c_p, \\ K(\varphi x, \varphi y, z, u) = K(x, y, z, u), \quad x, y \in D^c_p, \\ K(x, y, z, \xi) = 0. \end{array} \right.$$

Lemma 6 and (13) imply

Lemma 7. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type. Then the essential components of the Hermitian- like curvature tensor are $K_{\alpha\bar{\beta}\gamma\bar{\delta}}$, $K_{\circ\beta\gamma\bar{\delta}}$ and satisfy*

$$K_{\alpha\bar{\beta}\gamma\bar{\delta}} = -\partial_\alpha\partial_{\bar{\beta}}g_{\gamma\bar{\delta}} + g^{\lambda\bar{\mu}}(\partial_\alpha g_{\gamma\bar{\mu}})(\partial_{\bar{\beta}}g_{\lambda\bar{\delta}}) =$$

$$= R_{\alpha\bar{\beta}\gamma\bar{\delta}} + \frac{3}{2}i[(\nabla_\alpha d\Phi)_{\bar{\beta}\bar{\delta}\gamma} - (\nabla_{\bar{\beta}}d\Phi)_{\alpha\gamma\bar{\delta}}] - \frac{3}{2}[\Phi^{\lambda}_{\alpha\gamma}\partial_{\bar{\beta}}\Phi_{\lambda\bar{\delta}} - \Phi^{\bar{\mu}}_{\bar{\beta}\bar{\delta}}\partial_\alpha\Phi_{\gamma\bar{\mu}}],$$

$$K_{\circ\beta\gamma\bar{\delta}} = \partial_\circ\partial_\beta g_{\gamma\bar{\delta}} - g^{\lambda\bar{\mu}}(\partial_\circ g_{\lambda\bar{\delta}})(\partial_\beta g_{\gamma\bar{\mu}}) =$$

$$= -2R_{\circ\beta\gamma\bar{\delta}} - 3i(\nabla_\circ d\Phi)_{\beta\gamma\bar{\delta}} + \frac{9}{2}\Phi^{\lambda}_{\circ\beta}\Phi_{\lambda\gamma\bar{\delta}},$$

where R_{ABCD} are components of the Riemannian curvature tensor on M.

Definition 3. A linear connection D on an almost contact metric manifold $M(\varphi,\xi,\eta,g)$ is said to be a *characteristic connection* if D is a symmetric connection, preserving the structure tensor field φ, i.e. $T=0, D\varphi=0$ are valid identically.

It is well known that for any linear connection $\bar{D}$ with a torsion tensor field $\bar{T}$, the connection $\tilde{D} = \bar{D} - \frac{1}{2}\bar{T}$, associated with $\bar{D}$, is a symmetric linear connection having the same geodesy as $\bar{D}$.

Theorem 3. *Let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold. The connection $\overset{*}{D}$, associated with the Hermitian-like natural connection D on M, is a characteristic connection iff $M(\varphi,\xi,\eta,g)$ is of Hermitian type.*

Proof. Let $\overset{*}{D} = D - \frac{1}{2}T$ be a characteristic connection. This is equivalent to

$$T(x,\varphi y,z) + T(x,y,\varphi z) = 0. \tag{14}$$

In terms of the complex basis $\{Z_A\}_{A\in I\cup\bar{I}\cup I_\circ}$ (14) implies that $T_{\alpha\beta\bar{\gamma}}$ and $T_{\circ\beta\bar{\gamma}}$ are the essential components of the $\mathbf{C}$-linear extension of the Hermitian-like torsion tensor field T. Lemma 4 and Definition 2 imply $T_{\circ\beta\bar{\gamma}} = \frac{1}{2}T_{\beta\bar{\gamma}\circ} = \eta_{\beta\bar{\gamma}} = 0$. Now Lemma 5 ii) implies the vanishing of the essential components of N and $d\eta$, i.e. M is of Hermitian type.

Conversely let $M(\varphi,\xi,\eta,g)$ be an almost contact metric manifold of Hermitian type. Lemma 6ii) implies the vanishing of the components $T_{A\beta\gamma}$ and $T_{A\beta\circ}$. After the usual linearization one can obtain (14) and hence $\overset{*}{D}$ is a characteristic connection. □

In terms of local complex coordinate system on an almost contact metric manifold of Hermitian type, the essential components of $\overset{*}{D}$ satisfy

$$\overset{*}{D}{}^{\gamma}_{\alpha\beta} = \Gamma^{\gamma}_{\alpha\beta} = D^{\gamma}_{\alpha\beta} - \frac{3}{2}i\Phi^{\gamma}_{\alpha\beta} = g^{\gamma\bar{\sigma}}\partial_\alpha g_{\beta\bar{\sigma}}.$$

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Department of Mathematics
"Vasil Levsky" Higher Military School
5000 Veliko Tarnovo, Bulgaria

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